

Topic : Binomial Theorem

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1 to 8	(3 marks, 3 min.)	[24, 24]
Subjective Questions (no negative marking) Q.9, 10	(4 marks, 5 min.)	[8, 10]

- If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is
 (A) 6 (B) 9 (C) 12 (D) 24.
- For $2 \leq r \leq n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$
 (A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$
- In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then a/b equals:
 (A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$
- The sum $\sum_{i=0}^m \binom{10}{i} \binom{20}{m-i}$, (where $\binom{p}{q} = 0$, if $p < q$) is maximum when 'm' is:
 (A) 5 (B) 10 (C) 15 (D) 20
- Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is:
 (A) ${}^{12}C_6 + 3$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) ${}^{12}C_6 + 2$
- If ${}^{(n-1)}C_r = (k^2 - 3) {}^nC_{r+1}$, then an interval in which k lies is
 (A) $(2, \infty)$ (B) $(-\infty, -2)$ (C) $[-\sqrt{3}, \sqrt{3}]$ (D) $(\sqrt{3}, 2]$
- The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \dots + \binom{30}{20}\binom{30}{30}$ is :
 (A) $\binom{60}{20}$ (B) $\binom{30}{10}$ (C) $\binom{30}{15}$ (D) None of these



8. For $r = 0, 1, \dots, 10$, let A_r , B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to
- (A) $B_{10} - C_{10}$ (B) $A_{10}(B_{10}^2 - C_{10}A_{10})$ (C) 0 (D) $C_{10} - B_{10}$

9. Prove that $2^k \binom{n}{0} \binom{n}{k} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n}{2} \binom{n-2}{k-2} - \dots + (-1)^k \binom{n}{k} \binom{n-k}{0} = \binom{n}{k}$.

10. For any positive integer m, n (with $n \geq m$), let $\binom{n}{m} = {}^nC_m$. Prove that

$$\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \dots + \binom{m}{m} = \binom{n+1}{m+1}.$$

Hence or otherwise, prove that

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \dots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}.$$

Answers Key

1. (C)
2. (D)
3. (B)
4. (C)
5. (D)
6. (D)
7. (B)
8. (D)